

國立中山大學 111 學年度

碩士班暨碩士在職專班招生考試試題

科目名稱：離散數學【資工系碩士班甲組】

—作答注意事項—

考試時間：100 分鐘

- 考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- 答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張)。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

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題號：434004

※本科目依簡章規定「不可以」使用計算機(問答申論題)

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There are 8 problems in this test. Note that you should write down detailed steps for the solution to each problem; otherwise, no credits for that problem will be given.

1. [10%] Let S be a set of seven positive integers the maximum of which is at most 21. Prove that the sums of the elements in all the nonempty subsets of S cannot all be distinct.
2. [10%] Construct a state diagram for a finite state machine with $I = O = \{0,1\}$ that recognizes all strings in the language $\{0,1\}^*\{11\} \cup \{0,1\}^*\{01\}$.
3. [10%] Let $n \in \mathbb{Z}^+$ with $n > 1$, and let A be the set of positive integer divisors of n . Define the relation R on A by xRy if x (exactly) divides y . Determine how many ordered pairs are in the relation R when n is 6680.
4. [10%] Let n be a (fixed) positive integer, with $n \geq 5$. If $1 \leq t \leq \lfloor \frac{n}{2} \rfloor$, how many palindromes of n start with t ?
5. If a 26-digital ternary $(0,1,2)$ sequence is randomly generated, what is the probability that:
(a) [10%] It has an even number of 0's and an even number of 2's?
(b) [10%] It has an odd number of 0's, an odd number of 1's, and an even number of 2's.
6. (a) [10%] For $n \geq 1$, let a_n count the number of binary strings of length n , where there are no consecutive 0's. Find and solve a recurrence relation for a_n .
(b) [10%] For $n \geq 1$, let b_n count the number of binary strings of length n , where there are no consecutive 1's and the last bit of the string is 0. Find and solve a recurrence relation for b_n .
7. [10%] If a first case of COVID-19 is recorded in a country, let p_n denote the probability that a least one case is reported during the n th week after the first recorded case in case that the percentage of the population that has received at least two doses of the COVID-19 vaccine achieves 70%. The country records provide evidence that $p_n = p_{n-1} - (0.3)p_{n-2}$, where $n \geq 2$. Since $p_0 = 0$ and $p_1 = 1$, if the first case (of a new outbreak) is recorded on December 27, 2019, when did the probability for the occurrence of a new case decrease to less than 0.01 for the first time?
8. [10%] How many ways are there to place 20 marbles of the same size in six distinct jars if each marble is a different color?