

國立中山大學 107 學年度碩士暨碩士專班招生考試試題

科目名稱：離散數學【資工系碩士班甲組】

題號：434004

※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 1 頁第 1 頁

There are 6 problems in this test. Note that you should write down detailed steps for the solution to each problem; otherwise, no credits for that problem will be given.

- Find the coefficient of x^7 in each of the following formulas.
 - [10%] $((x^2-3x)/(1-x)^5)+3x^7+5$.
 - [10%] $-2/((x-1)(x-2))$.
- Consider the counting of the number of onto functions $H: A \rightarrow B$ where $|A| \geq |B|$.
 - [10%] Find the exponential generating function for the above counting such that the coefficient of $\frac{x^{|A|+2}}{|A|!}$ is the answer of the above counting.
 - [10%] Apply the above exponential generating function to find the answer of the above counting where $|A| = 10$ and $|B| = 3$.
- Let $\Sigma = \{0, 1\}$ be an alphabet and $A = \{1, 00, 10\}$ be a subset of Σ^* .
 - [10%] For each integer $n \geq 1$, let a_n be the number of strings in A^* of length n . Find and solve a recurrence relation for a_n .
 - [10%] For each integer $n \geq 1$, let b_n be the number of strings in A^* which are of length n and exactly divided by 2 when we regard each of the strings as a binary number. Find b_n .
- Consider the additive group $(\mathbb{Z}_8, +)$.
 - [5%] What is the order of 6? Why?
 - [5%] Find all generators of the group.
- Consider the multiplicative group $(\mathbb{Z}_{196}^*, \cdot)$.
 - [5%] What is the order of the group? Why?
 - [5%] Find the inverse of 25.
- Let $(\mathbb{Z} \times \mathbb{Z}, \oplus)$ be the group with $(a, b) \oplus (c, d) = (a+c+2, b+d-2)$ for any $(a, b), (c, d) \in \mathbb{Z} \times \mathbb{Z}$ where $a+c+2$ and $b+d-2$ are computed using ordinary addition and subtraction in $\mathbb{Z}$.
 - [2%] $3(4, 5) = (4, 5) \oplus (4, 5) \oplus (4, 5) = ?$
 - [5%] What is the identity of the group?
 - [5%] What is the inverse of (a, b) ?
 - [8%] Let $(G, +)$ be an additive group and let $h: \mathbb{Z} \times \mathbb{Z} \rightarrow G$ be a group homomorphism where $h(3, 2) = u$ and $h(-2, 6) = v$. Please express $h(18, 22)$ in terms of u and v .