

國立中山大學 105 學年度碩士暨碩士專班招生考試試題

科目名稱：離散數學【資工系碩士班甲組】

題號：434004

※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 1 頁第 1 頁

This test consists of 7 problems. No calculators are allowed. Write down detailed steps for the solution to each problem. Otherwise, no credits for that problem will be given.

1. (15) Show that for any positive integer n , there exists a *positive integer* m such that n divides m (namely, $m = kn$ for some integer k), and the decimal digits of m are only 0 and 1. For example, $n = 7$, $m = 1001$. In addition to show that this is true in general, use $n = 7$ and $n = 12$ to show how to find such m .
2. (15) Let N be the set of all positive integers, and $S = N \times N$, (namely, $S = \{(x, y) \mid x, y \in N\}$). Show that there is a bijection (one-one and onto) function $f : S \rightarrow N$, or show that no such function exists.
3. (15) Let $m > 1$ and $n > 1$ be two positive integers. Let $r(m, n)$ denotes the *maximum* number of rectangles defined by m horizontal lines and n vertical lines in a plane. Derive a formula for $r(m, n)$, and justify your answer. Note that rectangles may overlap. For example, let $m = 2$ and $n = 3$ () , $r(2, 3) = 3$, not 2.
4. (15) A *line segment* in R^2 is a line joining 2 different points in R^2 . Two lines intersect if they share a common point. Show that in any 6 line segments in R^2 , *either* there are 3 line segments in which any pair of them intersect *or* there are 3 line segments in which no pair of them intersect.
5. (10) Let the sequence of numbers $g_0, g_1, \dots, g_n, \dots$ be defined by $g_0 = 0$, $g_1 = 1$, and, for every $n > 1$, $g_n = g_{n-1} + g_{n-2}$. Solve g_n , in terms of n , by the method of generating functions.
6. (15) The girth of a graph G is the length of the shortest cycle in G . Let G be a simple graph with ν vertices, ϵ edges, and girth g . It is known that if G is planar, then $\epsilon \leq \frac{g}{g-2}(\nu - 2)$. Let K_5 be a complete graph with 5 vertices, and $K_{3,3}$ be a complete bipartite graph with 3 vertices in each partition. Draw K_5 and $K_{3,3}$, and show that they are not planar.
7. (15) Let L be the set of binary strings whose values are multiple of 7, (namely, $L = \{0, 111, 1110, 10101, \dots\}$). Design a finite state machine $M = (Q, \Sigma, \delta, q_0, F)$ to recognize the language L . Although no formal proof is required, you should explain why your machine M accepts only strings in L , and every binary string in L is accepted by M .