

國立中山大學 103 學年度碩士暨碩士專班招生考試試題

科目名稱：離散數學【資工系碩士班甲組】

題號：434004

※本科目依簡章規定「不可以」使用計算機

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There are 8 problems in this test. Note that you should write down detailed steps for the solution to each problem; otherwise, no credits for that problem will be given.

1. [10%] (a) [2%] What is the Well-Ordering Principle? (b) [2%] What is the Principle of Mathematical Induction? (c) [6%] Please use the Well-Ordering Principle to prove that the Principle of Mathematical Induction is true?
2. [15%] Let $X = \{x_1, x_2, \dots, x_m\}$ and $Y = \{y_1, y_2, \dots, y_n\}$ with $|X| = m$ and $|Y| = n$.
 - (a) [3%] How many functions $f: X \rightarrow Y$ satisfy $f(x_1) = f(x_m) = y_n$?
 - (b) [3%] Let $m \leq n$. How many functions $f: X \rightarrow Y$ are one-to-one?
 - (c) [3%] Let $m = n$. How many bijective functions $f: X \rightarrow Y$ satisfy $f(x_1) \neq y_1$?
 - (d) [3%] How many relations are there from X to Y ?
 - (e) [3%] How many relations R from X to Y satisfy $x_1 R y_1$ and $x_2 R y_2$?
3. [15%] Solve the following recurrence relations:
 - (a) [5%] $a_n + 3a_{n-1} + 2a_{n-2} = 0, n \geq 2, a_0 = 1, a_1 = 4$
 - (b) [5%] $a_n - 10a_{n-1} + 25a_{n-2} = 0, n \geq 2, a_0 = 1, a_1 = 0$
 - (c) [5%] $3a_n + 5a_{n-1} + 2a_{n-2} = 10n + 11, n \geq 2, a_0 = 10, a_1 = -10/3$
4. [10%] Let $I = \{0, 1\}$ be the input alphabet and $O = \{0, 1\}$ be the output alphabet of a finite state machine that recognizes all strings in the language $\{0, 1\}^* \{01\} \{0, 1\}^+$. Please draw a state diagram for the finite state machine after minimization.
5. [12%] Find the coefficient of x^n in each of the following formulas:
 - (a) [2%] $(2+3x)^{1000}$; (b) [4%] $(x^7 + x^8 + x^9 + \dots)^9$; (c) [6%] $\frac{4}{x^2 - 8x + 15}$.
6. [8%] Consider the number of the partitions of a positive integer into positive summands without regard to order. For example, $4 = 1+1+1+1 = 1+1+2 = 2+2 = 1+3$, so the integer 4 has five partitions. Find the generating function for the number of the partitions of a positive integer n into positive summands where each summand appears at most twice in each of the partitions. Please express your generating function in a closed form and indicate how to find the number of the partitions of n from your generating function.
7. [10%] Please compute $w = (995^{1010} \bmod 97)$, $x = (3866^{2402} \bmod 385)$, and $y = ((w-x) \bmod 17)$, and then output (w, x, y) .
8. [20%] Let p and q be two distinct prime numbers and $n = p \cdot q$ be the product of p and q . For any u_1, u_2 in $\mathbb{Z}_p^*$ and any v_1, v_2 in $\mathbb{Z}_q^*$, define $(u_1, v_1) * (u_2, v_2) = (u_1 * u_2 \bmod p, v_1 * v_2 \bmod q)$. (a) [5%] Please show that $(\mathbb{Z}_p^* \times \mathbb{Z}_q^*, *)$ is a commutative group. (b) [15%] Prove that $(\mathbb{Z}_p^* \times \mathbb{Z}_q^*, *)$ is isomorphic to $(\mathbb{Z}_n^*, *)$.