

國立成功大學

111學年度碩士班招生考試試題

編 號：202

系 所：電機資訊學院-資訊聯招

科 目：計算機數學

日 期：0219

節 次：第 3 節

備 註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

一、離散數學 (50%)

1. (10 points) How many bit string of length 20 contain at most

Nineteen 1s? Please expressed as $A^B + C$

(a) $A = \underline{\hspace{2cm}}$ (3 point)

(b) $B = \underline{\hspace{2cm}}$ (3 points)

(c) $C = \underline{\hspace{2cm}}$ (4 points)

2. (10 points) Let (u, v) be a minimum-weight edge in a connected graph G . Show that (u, v) belongs to some minimum spanning tree of G

3. (10 points) $A = \{5, 6, 7, 8\}$ 、 $B = \{u, v\}$ 、 $C = \{\}$

consider $a = |\text{Power Set}(A)|$

$b = |A \times B|$ ($\times$ is cross product)

$c = |\text{Power Set}(C)|$

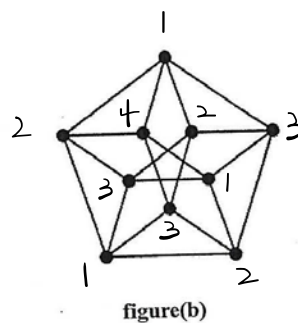
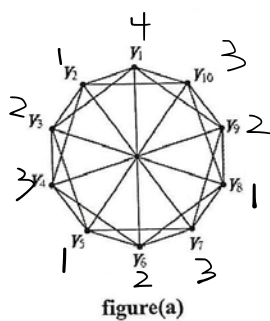
Please calculate $2^c + b + a$.

4. (a) (5 points) How many quadrilateral are determined by the vertices of a regular polygon of n sides? ($n \geq 4$)

- (b) (5 points) How many if no side of the polygon is to be a side of any quadrilateral? ($n \geq 8$)

Note: quadrilateral (四邊形)

5. (10 points) Calculate the chromatic number of figure(a) (5 points) and figure(b) (5 points). (not need to explain)



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二. Linear Algebra (50%)

6. Let $\mathbf{A}$ be a Hermitian matrix with eigenvalues $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ and orthonormal eigenvectors $\mathbf{u}_1, \dots, \mathbf{u}_n$. For any nonzero vector $\mathbf{x}$ in $\mathbb{R}^n$, the *Rayleigh quotient* $\rho(\mathbf{x})$ is defined by

$$\rho(\mathbf{x}) = \frac{\langle \mathbf{A}\mathbf{x}, \mathbf{x} \rangle}{\langle \mathbf{x}, \mathbf{x} \rangle} = \frac{\mathbf{x}^H \mathbf{A} \mathbf{x}}{\mathbf{x}^H \mathbf{x}}$$

- (a) If $\mathbf{x} = c_1 \mathbf{u}_1 + \dots + c_n \mathbf{u}_n$, show that

$$\rho(\mathbf{x}) = \frac{|c_1|^2 \lambda_1 + |c_2|^2 \lambda_2 + \dots + |c_n|^2 \lambda_n}{\|\mathbf{c}\|^2}$$

(10%)

- (b) Show that $\lambda_n \leq \rho(\mathbf{x}) \leq \lambda_1$. (5%)

- (c) Show that $\max_{\mathbf{x} \neq 0} \rho(\mathbf{x}) = \lambda_1$ and $\min_{\mathbf{x} \neq 0} \rho(\mathbf{x}) = \lambda_n$. (5%)

7. Let

$$\|\mathbf{A}\|_F = (\langle \mathbf{A}, \mathbf{A} \rangle)^{\frac{1}{2}} = \left(\sum_{i=1}^m \sum_{j=1}^n a_{ij}^2 \right)^{\frac{1}{2}} \quad \text{for } \mathbf{A} \in \mathbb{R}^{m \times n}.$$

The *trace* of an $n \times n$ matrix $\mathbf{C}$, denoted $\text{tr}(\mathbf{C})$, is the sum of its diagonal entries; that is,

$$\text{tr}(\mathbf{C}) = c_{11} + c_{22} + \dots + c_{nn}$$

If $\mathbf{A}$ and $\mathbf{B}$ are $m \times n$ matrices, show that

(a) $\|\mathbf{A}\|_F^2 = \text{tr}(\mathbf{A}^T \mathbf{A})$ (5%)

(b) $\|\mathbf{A} + \mathbf{B}\|_F^2 = \|\mathbf{A}\|_F^2 + 2\text{tr}(\mathbf{A}^T \mathbf{B}) + \|\mathbf{B}\|_F^2$ (5%)

8. Let $\mathbf{A}$ be a symmetric $n \times n$ matrix. Show that $e^{\mathbf{A}}$ is symmetric and positive definite. (10%)

9. Let $\mathbf{A}$ be a singular $n \times n$ matrix. Show that $\mathbf{A}^T \mathbf{A}$ is positive semidefinite, but not positive definite. (10%)